

Name of Test	Tests for	Statement
$n^{\text{th}}$ -term Test	Divergence	If $\lim_{n \rightarrow \infty} a_n \neq 0$ , then $\sum_{n=1}^{\infty} a_n$ diverges.
Integral Test	Con & Div	If $a_n = f(n)$ for some continuous function $f$ that is positive and decreasing, then $\sum_{n=1}^{\infty} a_n$ converges/diverges if $\int_1^{\infty} f(x) dx$ converges/diverges.
$p$ -Test	Con & Div	If $p \leq 1$ then $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges. If $p > 1$ then $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges.
Geometric Series	Con & Div	The series $\sum_{n=1}^{\infty} ax^n$ diverges if $ x  \geq 1$ and converges to $\frac{a}{1-x}$ if $ x  < 1$ . Note that no matter what $x$ is, we have the $n^{\text{th}}$ partial-sum $S_n = a \frac{1-x^{n+1}}{1-x}$ .
(Direct) Comparison Test	Con & Div	Let $0 \leq a_n \leq b_n$ for all $n$ . Then if $\sum_{n=1}^{\infty} b_n$ converges, $\sum_{n=1}^{\infty} a_n$ must converge as well. On the flipside, if $\sum_{n=1}^{\infty} a_n$ diverges then $\sum_{n=1}^{\infty} b_n$ diverges too.
Limit Comparison Test	Con & Div	If $a_n > 0$ and $b_n > 0$ and if $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L > 0$ , then $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ either both converge or both diverge.
Ratio Test	Con & Div	If $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} < 1$ , then $\sum_{n=1}^{\infty} a_n$ converges. Furthermore, if $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} > 1$ , then $\sum_{n=1}^{\infty} a_n$ diverges. If $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = 1$ , then the test is <i>inconclusive</i> .
Alternating Series Test	Convergence	If $0 < a_{n+1} < a_n$ (decreasing) and $\lim_{n \rightarrow \infty} a_n = 0$ , then $\sum_{n=1}^{\infty} (-1)^n a_n$ converges. Failing either of the first two criteria means the test is <i>inconclusive</i> .
Absolute Convergence	Convergence	If $\sum_{n=1}^{\infty}  a_n $ converges, then $\sum_{n=1}^{\infty} a_n$ converges.