Some Helpful Things

Math 101 February 25, 2019

Warning: Only studying this sheet will NOT help you with the exam. This is meant to be a supplemental reference sheet to aid with studying.

1 Some Exponential Equations

Equation	$y = a(b)^t$	$y = P(1+\frac{r}{n})^{nt}$	$y = Pe^{rt}$
Growth Factor	b	$(1+\frac{r}{n})^n$	e^r
(Effective) Growth Rate	$b-1$	$(1+\frac{r}{n})^n - 1$	$e^r - 1$
Nominal Growth Rate	n/a	r	r
Continuous Growth Rate	$\ln(b)$	not important	$\ln(e^r) = r$

Note that when we have the formula $y = a(b)^t$, the growth rate is $b-1$ and the *continuous* growth rate is $\ln(b)$. For **example**, suppose we had the formula $y = 3(0.75)^t$. Then the growth factor is 0.75 and the effective annual growth rate is $0.75 - 1 = -0.25 = -25\%$. The continuous growth rate is $\ln(0.75) \approx -0.2877 = -28.77\%$.

Half-life & **Doubling time:** If a value halves every k years, then the growth factor for the exponential equation is $\sqrt[k]{\frac{1}{2}}$. For example, if we have 70 grams of carbon-14 and it has a half-life of 8 years, then our growth factor is $\sqrt[8]{\frac{1}{2}}$ and our initial value is 70. Thus our equation for the amount of carbon-14 remaining is $y = 70\sqrt[8]{\frac{1}{2}}^{\,t}$. We can then find the annual growth rate and continuous growth rate by consulting the table above.

If a value doubles every k years, then the growth factor for the exponential equation is $\sqrt[k]{2}$. For example, if a population is 99 and it doubles every 50 years, then our formula for the population over time is $y = 99\sqrt[50]{2}^{\,t}$. We may again find the annual growth rate and continous growth rate by looking at the table above.

If we're given an exponential formula $a(b)^t$ and asked to find the doubling time, we can set $2 = b^t$ and solve for t. For example, say we're given the formula $500(1.8)^t$ and asked to find the doubling time. Then we may set $(1.8)^t = 2$ and solve for t. Solving for t gives us $t = \log_{1.8}(2) \approx 1.18$ years. To find the half-life, we simply set b^t equal to $\frac{1}{2}$ instead of 2.

2 Some Log Properties

1. $b^{\log_b(a)} = a$
2. $\log_b(b^a) = a$
3. $\log_b(a \cdot c) = \log_b(a) + \log_b(c)$ (equivalently, $\log_b(\frac{a}{c}) = \log_b(a) - \log_b(c)$)
4. $\log_b(a^c) = c \cdot \log_b(a)$
5. $\log_b(b) = 1$