$$
\begin{vmatrix} f_{xx} & f_{xy} & f_{xz} \\ f_{xy} & f_{yy} & f_{yz} \\ f_{xz} & f_{yz} & f_{zz} \end{vmatrix}.
$$

 $\leq k$ that $E \cong V(y^2z - x^3 + axz^2 + bz^3)$. Such a curve is called **Weierstrass form**, $\mathcal{L}_{\mathcal{D}}^{(b)}$ **Proposition 1.** Let $E \subseteq \mathbb{P}_k^2$ be an elliptic curve, $k = \mathbb{C}$. Then there exist an $a, b \in k$ such that $E \cong V(y^2z - x^3 + axz^2 + bz^3)$. Such a curve is called **Weierstrass form.** \mathcal{L}^2 and \mathcal{L}^2 and \mathcal{L}^2 and \mathcal{L}^2 matrices in \mathcal{L}^2 $S₂$ $S₂$ form

 \mathcal{L} . The 22M H27i radiation of \mathcal{L} and \mathcal{L} and 2tHz \mathcal{L} **Definition 2.** Let $E \subseteq \mathbb{P}_k^2$ be an elliptic curve in Weierstrass form. Then the *j*-invariant of E is $\frac{4a^3}{a^2b^2}$

$$
1728 \cdot \frac{4a^3}{4a^3 + 27b^2}.
$$

The j -invariant is an isomorphism invariant of elliptic curves.

Theorem K-. Let E be an elliptic curve with j-invariant j. Then the j-invariant of $\bar{\mathcal{E}}E$ is \mathbb{R}^+ . Let *E* be an emplic curve with *f*-invariant *f*. Then the *f*-invariant of \mathbb{R} *E*

$$
H(j) = \frac{(6912 - j)^2}{27j^2}.
$$

Of particular interest are the *j*-invariants that are periodic over H .

For instance, the following nine numbers satisfy $H^2(j) = j$. r^2 because, the following finite frampers satisfy $H'(J) = J$.

$$
j = 1728
$$

\n
$$
j = \frac{3456}{7} \left(-1 - 3i\sqrt{3}\right)
$$

\n
$$
j = \frac{3456}{7} \left(-1 + 3i\sqrt{3}\right)
$$

\n
$$
j = 3456 \left(5 - 3\sqrt{3}\right)
$$

\n
$$
j = 3456 \left(3\sqrt{3} + 5\right)
$$

\n
$$
j = -5184i\sqrt{3} - \frac{1}{2}\sqrt{-\frac{4514807808}{13} - \frac{1}{13}644972544i\sqrt{3}} + 1728
$$

\n
$$
j = -5184i\sqrt{3} + \frac{1}{2}\sqrt{-\frac{4514807808}{13} - \frac{1}{13}644972544i\sqrt{3}} + 1728
$$

\n
$$
j = 5184i\sqrt{3} - \frac{1}{2}\sqrt{-\frac{4514807808}{13} + \frac{644972544i\sqrt{3}}{13} + 1728}
$$

\n
$$
j = 5184i\sqrt{3} + \frac{1}{2}\sqrt{-\frac{4514807808}{13} + \frac{644972544i\sqrt{3}}{13} + 1728}
$$

\n
$$
j = 5184i\sqrt{3} + \frac{1}{2}\sqrt{-\frac{4514807808}{13} + \frac{644972544i\sqrt{3}}{13} + 1728}.
$$

Theorem K-. Let E be an elliptic curve where with j-invariant j periodic under H. Then there is an $n \in \mathbb{N}$ such that $\overline{\mathbf{z}}^n E = E$.

For example, if $j(E) = 1728$, then $\bar{\mathbf{z}}^2 E = E$. But if $j(E) = \frac{3456}{7}(-1 - 3i)$ √ For example, if $j(E) = 1728$, then $\bar{\mathcal{R}}^2 E = E$. But if $j(E) = \frac{3456}{7}(-1 - 3i\sqrt{3})$, then হ ${}^3E = E.$ $\frac{1}{\pi}$ $\Phi^2 E = E$ But if $i(E) = \frac{3456}{2}(-1 - 3i\sqrt{3})$ then \mathcal{L} $\mathcal{$

spanned by E and $\overline{\epsilon}E$. **Definition 3.** Let E be an elliptic curve. The **Hesse pencil** $\mathcal{P}(E)$ generated by E is

> permutes the isomorphic nbers of the riesse pend The key: G_{216} permutes the isomorphic fibers of the Hesse pencil.

 $\frac{1}{2}$ ir Bi $\frac{1}{2}$ ratio $\frac{1}{2}$ is a contract of the $\frac{1}{2}$ in $\frac{1}{2}$ is a contract by $\frac{1}{2}$ distribution of the $\frac{1}{2}$ subset of the $\frac{1}{2}$ distribution of the $\frac{1}{2}$ distribution of the $\frac{$ This can be known by studying the automorphism group of the Hesse pencil (a subgroup
 $\mathcal{L}_{\text{tot}}(\mathbb{R}^2)$) salled the Hesse masses. Along with the fact that \mathcal{A} the Henry moves \mathcal{A} less models the fact that a the Hesse group. Along with $\mathcal{L}(\mathbf{A} \cup (\mathbf{m}^2))$ is the H \mathbf{H}^* is a substitute of \mathbf{H}^* of $Aut(\mathbb{P}^2)$, called the Hesse group. Along with the fact that

$$
\overline{\mathfrak{r}}\alpha(E) = \alpha(\overline{\mathfrak{r}}E) \text{ for all } \alpha \in \text{Aut}(\mathbb{P}^2).
$$

The Hesse group is \mathcal{L} and \mathcal{L} because \mathcal{L} becomes \mathcal{L} The Hesse group is $\frac{1}{2}$

$$
G_{216} \cong \Gamma \rtimes \Delta
$$

where $\Gamma \cong (\mathbb{Z}/3\mathbb{Z})^2$ preserves each fibre and $\Delta \cong SL_2(\mathbb{F}_3)$ acts on Γ by linear representation. Specifically,

$$
\Delta = \left\langle \alpha = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \zeta_3 & \zeta_3^2 \\ 1 & \zeta_3^2 & \zeta_3 \end{pmatrix}, \beta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \zeta_3 & 0 \\ 0 & 0 & \zeta_3 \end{pmatrix} \right\rangle \le \text{Aut}(\mathbb{P}^2) = \text{PGL}(3)
$$

and

$$
\Gamma \rtimes \langle \alpha^2 \rangle = \ker(G_{216} \to \text{Aut}(\mathbb{P}^1)).
$$

Theorem K-. The number of orbits of size n under H is

$$
\frac{\sum_{d|n} \mu(d) 3^{n/d}}{n}.
$$

Sequence [A027376](https://oeis.org/A027376) in OEIS.