Math 107 Calculus II Name: ____________________

Final Exam: December 13, 2017 6–8pm Signature: ____________________

In the column for when your lecture class meets, circle the name of your instructor and the name of your recitation leader. The second row gives the time and days that your lecture class meets.

Harbourne 8:30 MW	Pitts 9:30 MW	Kelley 11:30 MW	Kelley 12:30 MW	DeVries 6:30 MW
Scott Gensler	Austin Eide	Leilani Pai	Michael DeBellevue	DeVries
Austin Eide	Dylan McKnight	Michael DeBellevue	Kelsey Wells	
Kelsey Wells	Leilani Pai	Nicole Buczkowski	Nicole Buczkowski	
Jacob Kettinger	Kelsey Wells		Jacob Kettinger	
Dylan McKnight				

Instructions

- Turn off all communication devices.
- During the exam, you may not use any calculators or electronic devices of any sort. Nor may you use any notes, texts, references, etc.
- To receive full credit for a problem, you must provide a correct answer **and a sufficient amount of work** so that it can be determined how you arrived at your answer.
- Be sure your copy of this exam has 10 pages (including this page) with 11 problems; the last two pages are blank (they are for scratch work).
- Good luck!

Problem	Points	Score
1	16	
2	20	
3	18	
4	16	
5	20	
6	18	
7	16	
8	18	
9	18	
10	20	
11	20	
Total	200	

1. (16 points) Let $g(x)$ and $h(x)$ be continuous functions. Assume $\int_a^b h(x)dx = 7$, $\int_b^c h(x)dx = -2$ and $\int_a^b g(x)dx = 4$. Fill in each box with a specific number to make a true statement.

(a) $\int_a^a h(x)dx = \boxed{}$

(b) $\int_b^a h(x)dx = \boxed{}$

(c) $\int_a^c h(x)dx = \boxed{}$

(d) $\int_a^b (2g(x) - h(x))dx = \boxed{}$

2. (20 points) Suppose $f(x)$ is a polynomial with values and derivatives as given in the following table:

x	-1	0	1	2	3	4	5
$f(x)$	8	6	2	7	4	9	1
$f'(x)$	7	3	5	-9	5	7	2

Use the table of values given above and integration by parts to find the exact value of $\int_1^4 xf''(x)\,dx$. (Some of the data given in the table might not be needed.) As always, please show your work, and show explicitly how you used integration by parts to evaluate the integral.

3. (18 points) Let $y = f(x)$ be continuous and differentiable. Consider the estimates $LEFT(100)$, $RIGHT(100)$, $TRAP(100)$, and $MID(100)$ for the definite integral $\displaystyle\int_0^1 f(x)\,dx$.

(a) Fill in the box to get a true statement: If $LEFT(100) = 7$ and $RIGHT(100) = 9$, then $TRAP(100) = \boxed{}$.

(b) If $f(x)$ is decreasing, put $LEFT(100)$, $RIGHT(100)$ and $\int_0^1 f(x)dx$ in different boxes to get a true statement,

$$\boxed{} \leq \boxed{} \leq \boxed{}$$

(c) If $f(x)$ is decreasing, put $LEFT(100)$, $RIGHT(100)$ and $MID(100)$ in different boxes to get a true statement,

$$\boxed{} \leq \boxed{} \leq \boxed{}$$

(d) If $f(x)$ is concave up, put $MID(100)$, $TRAP(100)$ and $\int_0^1 f(x)dx$ in different boxes to get a true statement:

$$\boxed{} \leq \boxed{} \leq \boxed{}$$

4. (16 points) Assume $0 \leq f(x) \leq g(x)$ are continuous functions. For each circle TRUE if it is True (i.e., it is always true), or circle FALSE if it is False (i.e., it is not always true). For each case where it is false, give explicit functions $f(x)$ and $g(x)$ showing it is false.

(a) If $\int_1^\infty f(x)dx$ diverges, then so does $\int_1^\infty g(x)dx$. TRUE FALSE

(b) If $\int_1^\infty f(x)dx$ converges, then so does $\int_1^\infty g(x)dx$. TRUE FALSE

(c) If $\int_1^\infty g(x)dx$ converges, then so does $\int_1^\infty f(x)dx$. TRUE FALSE

(d) If $\int_1^\infty g(x)dx$ diverges, then so does $\int_1^\infty f(x)dx$. TRUE FALSE

5. (20 points) Use the substitution $x = 2 + \sin(t)$ to evaluate $\displaystyle\int \frac{1}{\sqrt{4x - 3 - x^2}}dx$. (Show what you get for dx and what integral you get after making the substitution, and then evaluate that integral. Show all of your work; your work must justify your steps.)

6. (18 points) In this problem, you will use comparisons to study $\displaystyle\int_1^\infty \frac{e^x}{-1+xe^x}\,dx$.

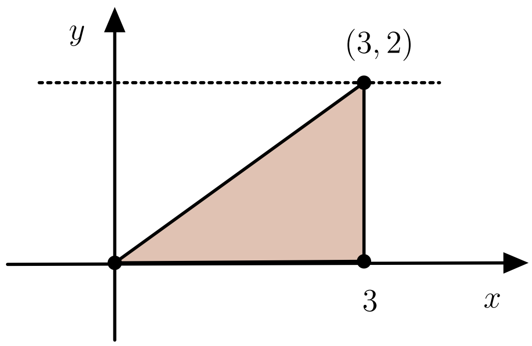
(a) Does $\displaystyle\int_1^\infty x^{-1}dx$ converge or diverge?

(b) Which of the following is true for $x \geq 1$? Circle the one which is true and justify your choice.

(i) $\displaystyle\frac{e^x}{-1+xe^x} < x^{-1}$ (ii) $\displaystyle x^{-1} < \frac{e^x}{-1+xe^x}$

(c) Based on your answers to (a) and (b), do you conclude that $\displaystyle\int_1^\infty \frac{e^x}{-1+xe^x}dx$ converges, diverges or that (a) and (b) do not provide enough information to say.

7. (16 points) Express as an integral the volume obtained by revolving the triangular shaded region in the figure below about the line $y = 2$ (shown as a dotted line in the graph below). You do not need to evaluate the integral. Note that the line given by the hypotenuse of the triangle is $y = \dfrac{2}{3}x$.



8. (18 points) Consider the sequence s_n given for $n \geq 1$ by $\frac{1}{2}, \frac{-1}{4}, \frac{1}{6}, \frac{-1}{8}, \frac{1}{10}, \ldots$. Thus the sign alternates, the numerator is always ± 1, and the denominator is always the next even number.

(a) Find a general expression for the nth term of the sequence, s_n: $\boxed{}$

(b) Determine whether the sequence is bounded, and if so give specific upper and lower bounds. If not, explain why it is not bounded.

(c) Determine whether the sequence is monotone, and if not, explain why it is not monotone.

9. (18 points) The integral test says that if $f(x)$ is a function which is continuous, positive and decreasing for $x \geq 1$, and if $a_n = f(n)$ for $n \geq 1$, then $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)dx$ either both converge or both diverge.

(a) The series $\sum_{n=0}^{\infty} a_n = 3 + \frac{1}{2} + \frac{3}{4} + \frac{1}{8} + \frac{3}{16} + \cdots$ has $a_n = \frac{2+\cos(n\pi)}{2^n}$ for $n \geq 0$. It converges, and we have $a_n = f(n)$ for $f(x) = \frac{2+\cos(x\pi)}{2^x}$, but you cannot use the integral test to justify why the series converges. Explain why the integral test does not apply.

(b) The series $\sum_{n=1}^{\infty} a_n = \frac{-1}{1} + \frac{-1}{2^2} + \frac{-1}{3^2} + \frac{-1}{4^2} + \cdots$ has $a_n = \frac{-1}{n^2}$ for $n \geq 1$. It converges, and we have $a_n = f(n)$ for $f(x) = \frac{-1}{x^2}$, but you cannot use the integral test to justify why the series converges. Explain why the integral test does not apply.

(c) Consider the series $e^{-1} + e^{-2} + e^{-3} + \cdots + e^{-n} + \cdots$. Does the integral test apply to this series? If not, why not? If so, indicate what function $f(x)$ you would use to apply the integral test to this series, explain why this $f(x)$ satisfies each of the three criteria needed, and indicate what the integral test tells you about convergence or divergence of this series.

10. (20 points) Circle TRUE or FALSE as appropriate for each of the following statements and for statements (a-d) indicate a convergence test that can be used to justify your conclusion.

(a) $\sum_{n\geq 1} \frac{(-1)^n}{\sqrt{n}}$ is absolutely convergent: TRUE FALSE | test: |

(b) $\sum_{n\geq 1} \frac{n}{n^3+1}$ diverges: TRUE FALSE | test: |

(c) $\sum_{n\geq 1} \frac{1}{n^3}$ converges: TRUE FALSE | test: |

(d) $\sum_{n\geq 1} \frac{(-1)^n}{n}$ converges: TRUE FALSE | test: |

(e) $-1 < \sum_{n\geq 1} \frac{(-1)^n}{n^2}$: TRUE FALSE

11. (20 points) Circle TRUE or FALSE, as appropriate, for each of the following.

(a) TRUE FALSE The Taylor series for $\sin(x)$ about $x = \pi$ is

$$(x-\pi) - \frac{(x-\pi)^3}{3!} + \frac{(x-\pi)^5}{5!} - \cdots$$

(b) TRUE FALSE The Taylor series for $x^3 \sin(x)$ about $x = 0$ has only odd powers.

(c) TRUE FALSE To find the Taylor series for $e^x + \cos(x)$ about any point, add the Taylor series for e^x and $\cos(x)$ about that point.

(d) TRUE FALSE The Taylor series about $x = 0$ for $\frac{1}{(1-x)^2}$ is the derivative of the Taylor series for $\frac{1}{1-x}$ about $x = 0$.

(e) TRUE FALSE $\sin(1) < 1 - \frac{1}{6}$

This page is for scratch work in case you need it.

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