Let S_5 be the projective plane $\mathbb{P}^2$ blown up at 5 points $P_1, \ldots, P_5$. Define $E_i = p^{-1}(P_i)$ for all $1 \leq i \leq 5$ where p is the blowup map. Let L be a generic line in $\mathbb{P}^2$ and define $E_0 = p^{-1}(L)$.

Then define $R_0 = E_0 - E_1 - E_2 - E_3$ and $R_i = E_i - E_{i+1}$ for all $1 \leq i \leq 4$. Note then that $R_i.R_i = -2$ and $R_i.K_5 = 0$ (where K_5 is the canonical divisor $-3E_0 + E_1 + E_2 + E_3 + E_4 + E_5$ on S_5) for all i.

Then define $s_i : \mathrm{Pic}S_5 \to \mathrm{Pic}S_5$ by $s_i(x) = x + (x.R_i)R_i$. These s_i generate the Weyl group W_5. The orbits of all the R_i form a root system $\Phi_5 = \bigcup_{i=0}^{4} W_5 \cdot R_i \subseteq \mathrm{Pic}S_5 \otimes_{\mathbb{Z}} \mathbb{R}$. The R_i are the simple roots of Φ_5.

We can then form a Dynkin diagram of the root system, where each vertex represents a simple root R_i and two vertices are connected if their respective simple roots are not orthogonal (their intersection product is not 0). The Dynkin diagram for Φ_5 is known as D_5:

$$\begin{tikzcd}
 & & R_0 \arrow[d, dash] & \\
R_1 \arrow[r, dash] & R_2 \arrow[r, dash] & R_3 \arrow[r, dash] & R_4
\end{tikzcd}.$$

A Coxeter element of W_5 is a product of $s_0, \ldots, s_4$ taken one at a time in any order. Since all Coxeter elements are conjugate, they have the same eigenvalues. For example, let us look at the Coxeter element $s = s_0 s_1 s_2 s_3 s_4$. Then we can construct the following table:

x	$s(x)$
R_0	$R_1 + R_2 + R_3$
R_1	R_2
R_2	$R_0 + R_3$
R_3	R_4
R_4	$-R_0 - R_1 - R_2 - R_3 - R_4$

thereby giving us the following matrix:

$$s = \begin{pmatrix} 0 & 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 1 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix}.$$

Then $\det(s) = -1$ and the five eigenvalues of s are $-1, \pm\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$. These all have a magnitude of 1, and so the spectral radius of s- defined to be the largest magnitude of the eigenvalues of s- is 1. Thus s has an entropy- defined as the logarithm of its spectral radius- of 0.

Next let us blow up $\mathbb{P}^2$ at 10 points to get the surface S_{10}, with R_i and s_i defined similarly as before. Then we can contruct the following Dynkin diagram known as E_{10} on Φ_{10}:

$$\begin{array}{ccccccccccccccccc} & & & & R_0 & & & & & & & & & & & & \\ & & & & | & & & & & & & & & & & & \\ R_1 & — & R_2 & — & R_3 & — & R_4 & — & R_5 & — & R_6 & — & R_7 & — & R_8 & — & R_9 \end{array}.$$

Note that $R_i.R_j = 1$ if $i \neq j$ and R_i and R_j are connected, $R_i.R_i = -2$ and $R_i.R_j = 0$ if $i \neq j$ and R_i and R_j are not connected. This gives us a piecewise formula for each s_i:

$$s_i(R_j) = \begin{cases} R_j + R_i & i \neq j \text{ and } R_i \text{ and } R_j \text{ are connected} \\ -R_i & i = j \\ R_j & i \neq j \text{ and } R_i \text{ and } R_j \text{ are not connected} \end{cases}.$$

This helps in looking at the Coxeter element $r = s_0 s_1 s_2 s_3 s_4 s_5 s_6 s_7 s_8 s_9 \in W_{10}$. We have the following table:

x	$r(x)$
R_0	$R_1 + R_2 + R_3$
R_1	R_2
R_2	$R_0 + R_3$
R_3	R_4
R_4	R_5
R_5	R_6
R_6	R_7
R_7	R_8
R_8	R_9
R_9	$-R_0 - R_1 - R_2 - R_3 - R_4 - R_5 - R_6 - R_7 - R_8 - R_9$

which gives us the following matrix representation:

$$r = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}.$$

We have $\det(r) = 1$ and the characteristic polynomial of r is

$$\chi(r) = x^{10} + x^9 - x^7 - x^6 - x^5 - x^4 - x^3 + x + 1,$$

known as Lehmer's polynomial. The maximum magnitude of a root of Lehmer's polynomial is known as Lehmer's number $\lambda_L \approx 1.17628081$. Therefore r has spectral radius $\lambda_L > 1$, and so r has positive entropy.

Compare this with the Coxeter element $t = s_0s_1s_2s_3s_4s_5s_6s_7s_8 \in W_9$. We have the matrix representation

$$t = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}.$$

The characteristic polynomial of t is

$$\chi(t) = -x^9 - x^8 + x^6 + x^5 + x^4 + x^3 - x - 1 = -(x-1)^2(x^7 + 3x^6 + 5x^5 + 6x^4 + 6x^3 + 5x^2 + 3x + 1).$$

We have the 8 eigenvalues of t are 1 (this one occurs with multiplicity 2, hence why there are 8 eigenvalues and not 9), -1, $\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$, $-\frac{2\sqrt{5}+2}{8} \pm i\frac{\sqrt{2\sqrt{5}+2}\sqrt[4]{5} - \sqrt{2\sqrt{5}+2}\sqrt[4]{125}}{8}$, and $\frac{\sqrt{5}-1}{4} \pm i\frac{\sqrt{2\sqrt{5}+2}\sqrt[4]{5}}{4}$. All of these numbers have a magnitude of 1 (I checked), and so t has a spectral radius of 1, and so the entropy of t is 0.

Let us now return to

$$r = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}$$

for a moment. Let us add a 1 in the corner to denote the preservation of k. We get

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Our change of basis matrix from the R_i's to the E_i's is

$$B = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix}$$

and

$$B^{-1} = \begin{pmatrix} 10 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \\ 7 & 3 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 14 & 5 & 5 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 \\ 21 & 7 & 7 & 7 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ 18 & 6 & 6 & 6 & 6 & 5 & 5 & 5 & 5 & 5 & 5 \\ 15 & 5 & 5 & 5 & 5 & 5 & 4 & 4 & 4 & 4 & 4 \\ 12 & 4 & 4 & 4 & 4 & 4 & 4 & 3 & 3 & 3 & 3 \\ 9 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 2 & 2 & 2 \\ 6 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 1 \\ 3 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 3 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

So

$$BAB^{-1} = \begin{pmatrix} 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

From this we can see that the map corresponds to a degree-2 birational map on $\mathbb{P}^2$.

In the case of blowing up 9 points, we get the change of basis matrix

$$\begin{pmatrix}
0 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\
0 & \frac{2}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\
0 & \frac{1}{3} & \frac{1}{3} & \frac{-2}{3} & \frac{-2}{3} & \frac{-2}{3} & \frac{-2}{3} & \frac{-2}{3} & \frac{-2}{3} & \frac{-2}{3} \\
0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 \\
0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
1 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3}
\end{pmatrix}$$

$$= \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0
\end{pmatrix}^{-1}$$

converting between the bases $\{\ell, e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9\}$ and $\{r_0, r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8, r_9, \ell\}$. Thus we can write

$$\begin{aligned}
\ell &= \ell \\
e_1 &= -\frac{1}{3}r_0 + \frac{2}{3}r_1 + \frac{1}{3}r_2 + \frac{1}{3}\ell \\
e_2 &= -\frac{1}{3}r_0 - \frac{1}{3}r_1 + \frac{1}{3}r_2 + \frac{1}{3}\ell \\
e_3 &= -\frac{1}{3}r_0 - \frac{1}{3}r_1 - \frac{2}{3}r_2 + \frac{1}{3}\ell \\
e_4 &= -\frac{1}{3}r_0 - \frac{1}{3}r_1 - \frac{2}{3}r_2 - r_3 + \frac{1}{3}\ell \\
e_5 &= -\frac{1}{3}r_0 - \frac{1}{3}r_1 - \frac{2}{3}r_2 - r_3 - r_4 + \frac{1}{3}\ell \\
e_6 &= -\frac{1}{3}r_0 - \frac{1}{3}r_1 - \frac{2}{3}r_2 - r_3 - r_4 - r_5 + \frac{1}{3}\ell \\
e_7 &= -\frac{1}{3}r_0 - \frac{1}{3}r_1 - \frac{2}{3}r_2 - r_3 - r_4 - r_5 - r_6 + \frac{1}{3}\ell \\
e_8 &= -\frac{1}{3}r_0 - \frac{1}{3}r_1 - \frac{2}{3}r_2 - r_3 - r_4 - r_5 - r_6 - r_7 + \frac{1}{3}\ell \\
e_9 &= -\frac{1}{3}r_0 - \frac{1}{3}r_1 - \frac{2}{3}r_2 - r_3 - r_4 - r_5 - r_6 - r_7 - r_8 + \frac{1}{3}\ell
\end{aligned}$$

But perhaps a more useful basis to use is $\langle r_0, r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8, e_7\rangle = K^\perp \oplus \langle e_7\rangle$. I will single out e_7 because in the quasielliptic fibration I'm thinking of, p_7 is the only point that is blown up once instead of twice. We get

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 3 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 3 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}^{-1}$$

which makes use of only integers.

Thus we can rewrite

$$\begin{aligned} \ell &= r_0 + r_1 + 2r_2 + 3r_3 + 3r_4 + 3r_5 + 3r_6 + 3e_7 \\ e_1 &= r_1 + r_2 + r_3 + r_4 + r_5 + r_6 + e_7 \\ e_2 &= r_2 + r_3 + r_4 + r_5 + r_6 + e_7 \\ e_3 &= r_3 + r_4 + r_5 + r_6 + e_7 \\ e_4 &= r_4 + r_5 + r_6 + e_7 \\ e_5 &= r_5 + r_6 + e_7 \\ e_6 &= r_6 + e_7 \\ e_7 &= e_7 \\ e_8 &= -r_7 + e_7 \\ e_9 &= -r_7 - r_8 + e_7 \end{aligned}$$

The sixteen (-2)-curves we get are as follows:

$$\begin{aligned}
\ell - e_1 - e_2 - e_7 \\
2\ell - e_3 - e_4 - e_5 - e_6 - e_8 - e_9 \\
\ell - e_3 - e_4 - e_7 \\
2\ell - e_1 - e_2 - e_5 - e_6 - e_8 - e_9 \\
\ell - e_5 - e_6 - e_7 \\
2\ell - e_1 - e_2 - e_3 - e_4 - e_8 - e_9 \\
\ell - e_7 - e_8 - e_9 \\
2\ell - e_1 - e_2 - e_3 - e_4 - e_5 - e_6 \\
e_1 - e_2 \\
3\ell - 2e_1 - e_3 - e_4 - e_5 - e_6 - e_7 - e_8 - e_9 \\
e_3 - e_4 \\
3\ell - e_1 - e_2 - 2e_3 - e_5 - e_6 - e_7 - e_8 - e_9 \\
e_5 - e_6 \\
3\ell - e_1 - e_2 - e_3 - e_4 - 2e_5 - e_7 - e_8 - e_9 \\
e_8 - e_9 \\
3\ell - e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - 2e_8.
\end{aligned}$$

Writing these in our new basis $\langle r_0, r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8, e_7\rangle$ yields

$$r_0 + r_3 + r_4 + r_5 + r_6 \tag{1}$$
$$2r_0 + 2r_1 + 4r_2 + 5r_3 + 4r_4 + 3r_5 + 2r_6 + 2r_7 + r_8 \tag{2}$$
$$r_0 + r_1 + 2r_2 + 2r_3 + r_4 + r_5 + r_6 \tag{3}$$
$$2r_0 + r_1 + 2r_2 + 4r_3 + 4r_4 + 3r_5 + 2r_6 + 2r_7 + r_8 \tag{4}$$
$$r_0 + r_1 + 2r_2 + 3r_3 + 3r_4 + 2r_5 + r_6 \tag{5}$$
$$2r_0 + r_1 + 2r_2 + 3r_3 + 2r_4 + 2r_5 + 2r_6 + 2r_7 + r_8 \tag{6}$$
$$r_0 + r_1 + 2r_2 + 3r_3 + 3r_4 + 3r_5 + 3r_6 + 2r_7 + r_8 \tag{7}$$
$$2r_0 + r_1 + 2r_2 + 3r_3 + 2r_4 + r_5 \tag{8}$$
$$r_1 \tag{9}$$
$$3r_0 + r_1 + 4r_2 + 6r_3 + 5r_4 + 4r_5 + 3r_6 + 2r_7 + r_8 \tag{10}$$
$$r_3 \tag{11}$$
$$3r_0 + 2r_1 + 4r_2 + 5r_3 + 5r_4 + 4r_5 + 3r_6 + 2r_7 + r_8 \tag{12}$$
$$r_5 \tag{13}$$
$$3r_0 + 2r_1 + 4r_2 + 6r_3 + 5r_4 + 3r_5 + 3r_6 + 2r_7 + r_8 \tag{14}$$
$$r_8 \tag{15}$$
$$3r_0 + 2r_1 + 4r_2 + 6r_3 + 5r_4 + 4r_5 + 3r_6 + 2r_7 \tag{16}$$

Notice that there is no e_7 vector present because all of these elements are in $K^{\perp}$. Now we want to take the free $\mathbb{Z}$-module $K^{\perp} = \langle r_0, r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8\rangle$ and mod out by the space generated by these sixteen (-2)-curves. Modding out by $\langle r_1, r_3, r_5, r_8\rangle$ yields $\langle r_0, r_2, r_4, r_6, r_7\rangle$. We then get from (1) and (3): $r_0 + r_4 + r_6 = r_0 + 2r_2 + r_4 + r_6 = 0$, and so $2r_2 = 0$ in the quotient.

From line (5) we also get $r_0 + 3r_4 + r_6 = 0$, and since we know $r_0 + r_4 + r_6 = 0$ from before, we get $2r_4 = 0$.

From line (4) we know that $2r_0+2r_6+2r_7 = 0$. From line (7) we get $r_0+r_4+3r_6+2r_7 = 0$, but since $r_0 + r_4 + r_6 = 0$, we get $2r_6 + 2r_7 = 0$. Since $2r_0 + 2r_6 + 2r_7 = 2r_6 + 2r_7 = 0$, we know $2r_0 = 0$.

From line (2), we know $2r_0+4r_4+2r_6+2r_7 = 0 = 2(r_0+r_4+r_6)+2r_4+2r_7 = 0+0+2r_7$. Thus $2r_7 = 0$ and since $2r_7 + 2r_6 = 0$, we know $2r_6 = 0$.

Thus our group is $\langle r_0, r_2, r_4, r_6, r_7 | 2r_{0\le i\le 7}, r_0 + r_4 + r_6\rangle \cong (\mathbb{Z}/2\mathbb{Z})^{\oplus 4}$. We can choose representative elements:

$$\begin{aligned}
&0\\
r_0 &= \ell - e_1 - e_2 - e_3\\
r_2 &= e_2 - e_3\\
r_4 &= e_4 - e_5\\
r_6 &= e_6 - e_7\\
r_7 &= e_7 - e_8\\
r_0 + r_2 &= \ell - e_1 - 2e_3\\
r_0 + r_7 &= \ell - e_1 - e_2 - e_3 + e_7 - e_8\\
r_2 + r_4 &= e_2 - e_3 + e_4 - e_5\\
r_2 + r_6 &= e_2 - e_3 + e_6 - e_7\\
r_2 + r_7 &= e_2 - e_3 + e_7 - e_8\\
r_4 + r_7 &= e_4 - e_5 + e_7 - e_8\\
r_6 + r_7 &= e_6 - e_8\\
r_0 + r_2 + r_7 &= \ell - e_1 - 2e_3 + e_7 - e_8\\
r_2 + r_4 + r_7 &= e_2 - e_3 + e_4 - e_5 + e_7 - e_8\\
r_2 + r_6 + r_7 &= e_2 - e_3 + e_6 - e_8
\end{aligned}$$

The sixteen (-1)-curves I found are:

$$
\begin{aligned}
&e_2\\
&e_4\\
&e_6\\
&e_7\\
&e_9\\
&\ell - e_1 - e_3\\
&\ell - e_1 - e_5\\
&\ell - e_3 - e_5\\
&\ell - e_1 - e_8\\
&\ell - e_3 - e_8\\
&\ell - e_5 - e_8\\
&2\ell - e_1 - e_3 - e_5 - e_7 - e_8\\
&2\ell - e_1 - e_3 - e_5 - e_8 - e_9\\
&2\ell - e_1 - e_2 - e_3 - e_5 - e_8\\
&2\ell - e_1 - e_3 - e_4 - e_5 - e_8\\
&2\ell - e_1 - e_3 - e_5 - e_6 - e_8
\end{aligned}
$$

The other potential candidates for (-1)-curves are imposters because they meet some (-2)-curves negatively. For example, $(2\ell-e_1-e_2-e_5-e_6-e_8).(2\ell-e_1-e_2-e_5-e_6-e_8-e_9) = -1$.