The canonical divisor K of $\mathbb{P}^2$ can be computed as $\operatorname{div}(\mathsf{d}f \wedge \mathsf{d}g)$, where f and g are rational functions on $\mathbb{P}^2$ in variables X, Y, and Z. Let $f = x := \frac{X}{Z}$ and let $g = y := \frac{Y}{Z}$, and $\omega = \mathsf{d}x \wedge \mathsf{d}y$.

First we must compute

$$-\operatorname{ord}_P\left(\frac{\mathsf{d}f_P \wedge \mathsf{d}g_P}{\mathsf{d}x \wedge \mathsf{d}y}\right),$$

where f_P and g_P generate $\mathfrak{m}_P$ (that is, f_P and g_P are linearly independent and $\operatorname{ord}_P(f_P) = \operatorname{ord}_P(g_P) = 1$). Then

$$\operatorname{div}(\omega) = \sum_{P \in \mathbb{P}^2} -\operatorname{ord}_P\left(\frac{\mathsf{d}f_P \wedge \mathsf{d}g_P}{\mathsf{d}x \wedge \mathsf{d}y}\right).$$

First let $P = (a, b, c)$ with $c \neq 0$. Then we can take $f_P = cx - a$ and $g_P = cy - b$. Then

$$-\operatorname{ord}_P\left(\frac{\mathsf{d}(cx-a) \wedge \mathsf{d}(cy-b)}{\mathsf{d}x \wedge \mathsf{d}y}\right) = -\operatorname{ord}_P\left(\frac{c\mathsf{d}x \wedge c\mathsf{d}y}{\mathsf{d}x \wedge \mathsf{d}y}\right) = -\operatorname{ord}_P(c^2) = 0.$$

Now let $Q = (a, b, 0)$ be on the line $Z = 0$, with $a \neq 0$. Then we can take $f_Q = \frac{y}{x} - \frac{b}{a}$ and $g_P = \frac{1}{x} = \frac{Z}{X}$. (Note: we cannot take f_Q or g_Q to be $bx - ay$, because $\operatorname{ord}_Q(bx - ay) = 0$, not 1. Recall $bx - ay = \frac{bX - aY}{Z}$. Instead we have $f_Q = \frac{aY - bX}{aX}$.) Then

$$\begin{aligned}
-\operatorname{ord}_Q\left(\frac{\mathsf{d}\left(\frac{y}{x} - \frac{b}{a}\right) \wedge \mathsf{d}\left(\frac{1}{x}\right)}{\mathsf{d}x \wedge \mathsf{d}y}\right) &= -\operatorname{ord}_Q\left(\frac{\mathsf{d}\left(\frac{y}{x}\right) \wedge \mathsf{d}\left(\frac{1}{x}\right)}{\mathsf{d}x \wedge \mathsf{d}y}\right) \\
= -\operatorname{ord}_Q\left(\frac{(x^{-1}\mathsf{d}y - x^{-2}y\mathsf{d}x) \wedge -x^{-2}\mathsf{d}x}{\mathsf{d}x \wedge \mathsf{d}y}\right) = -\operatorname{ord}_Q\left(\frac{x^{-1}\mathsf{d}y \wedge x^{-2}\mathsf{d}x}{\mathsf{d}x \wedge \mathsf{d}y}\right) &= -\operatorname{ord}_Q(-x^{-3}) \\
&= \operatorname{ord}_Q(x^3) = \operatorname{ord}_Q\left(\frac{X^3}{Z^3}\right) = -3.
\end{aligned}$$

Finally, let R be the point $(0, 1, 0)$. Then we can take $f_R = \frac{1}{y}$ and $g_R = \frac{x}{y}$. In the same kind of computation as for Q, we find

$$-\operatorname{ord}_R\left(\frac{\mathsf{d}f_R \wedge \mathsf{d}g_R}{\mathsf{d}x \wedge \mathsf{d}y}\right) = -3.$$

Then we see that for $P \in \mathbb{P}^2$, we have

$$-\operatorname{ord}_P(\omega) = \begin{cases} 0 & P \notin V(Z) \\ -3 & P \in V(Z) \end{cases}.$$

Therefore $K = \operatorname{div}(\omega) = -3L$, where L is the line $V(Z)$.