Recall that the coordinate ring of the affine plane $\mathbb{A}_k^2$ blown up at the point $(0,0)$ is $k[x,y,t,u]/(xt-yu)$.

Now consider the projective plane $\mathbb{P}_k^2$ blown up at the point (p_0,p_1,p_2), and call this surface $\mathbb{B}$. By projective duality, let $(\ell_0,\ell_1,\ell_2)\in(\mathbb{P}^2)^*\cong\mathbb{P}^2$ be a line in $\mathbb{P}^2$.

Then

$$\mathbb{B}=\{(x_0,x_1,x_2),(\ell_0,\ell_1,\ell_2)\in\mathbb{P}^2\times(\mathbb{P}^2)^*: p_0\ell_0+p_1\ell_1+p_2\ell_2=0, x_0\ell_0+x_1\ell_1+x_2\ell_2=0\}.$$

Then $\mathbb{B}\subseteq\mathbb{P}^2\times\mathbb{P}^2\hookrightarrow\mathbb{P}^8$ under the Segre embedding

$$((x_0,x_1,x_2),(\ell_0,\ell_1,\ell_2))\mapsto(x_0\ell_0,x_0\ell_1,x_0\ell_2,x_1\ell_0,x_1\ell_1,x_1\ell_2,x_2\ell_0,x_2\ell_1,x_2\ell_2).$$

Let us blow up $\mathbb{P}^2$ at the point $(p_0,p_1,p_2)=(0,0,1)$. Then we look at lines of the form $0\ell_0+0\ell_1+1\ell_2=0$ and so $\ell_2=0$.

Thus we can embed $\mathbb{B}$ into $\mathbb{P}^5$, with elements of the form

$$(x_0\ell_0,x_0\ell_1,x_1\ell_0,x_1\ell_1,x_2\ell_0,x_2\ell_1)=(w_0,w_1,w_2,w_3,w_4,w_5).$$

The w_i's satisfy the relation $w_0w_5=w_1w_4$ and $w_2w_5=w_3w_4$. This implies that $w_0w_3w_4w_5=w_1w_2w_4w_5$ and since the coordinate ring of the image of $\mathbb{P}^2\times\mathbb{P}^2$ in $\mathbb{P}^5$ is an integral domain (since the image is an irreducible algebraic variety) we can apply cancellation and conclude $w_0w_3=w_1w_2$ as well. Therefore $w_0w_3-w_1w_2\in(w_0w_5-w_1w_4,w_2w_5-w_3w_4)$. Thus the coordinate ring of the image of $\mathbb{P}^2\times\mathbb{P}^2$ in $\mathbb{P}^5$ is

$$k[W_0,W_1,W_2,W_3,W_4,W_5]/(W_0W_5-W_1W_4,W_2W_5-W_3W_4).$$

Recall that our x_i's and ℓ_i's must satisfy the relation $x_0\ell_0+x_1\ell_1=0$ in $\mathbb{B}$, and therefore our w_i's must satisfy the relation $w_0+w_3=0$. Therefore the coordinate ring of our blowup $\mathbb{B}$ is

$$k[W_0,W_1,W_2,W_3,W_4,W_5]/(W_0W_5-W_1W_4,W_2W_5-W_3W_4,W_0+W_3).$$

This implies $W_0=-W_3$ in the quotient and so this is isomorphic to

$$k[W_1,W_2,W_3,W_4,W_5]/(W_3W_5+W_1W_4,W_2W_5-W_3W_4).$$

Relabeling the letters gives us the coordinate ring for $\mathbb{B}$:

$$k[x,y,z,t,u]/(zu+xt,yu-zt).$$

Also the coordinate ring of $\mathbb{P}^3$ blown up at the point $(0,0,0,1)$ is

$$k[x_1,\dots,x_{11}]/(x_1x_5-x_2x_4,x_1x_6+x_4x_7+x_7x_8,x_1x_8-x_2x_7,$$
$$x_1x_9+x_4x_{10}+x_8x_{10},x_2x_3+x_4x_5+x_5x_8,x_2x_9+x_4x_{11}+x_8x_{11}).$$

Here

$$\mathbb{B}(\mathbb{P}^3,(p_0,p_1,p_2,p_3))=\left\{((x_0,x_1,x_2,x_3),(\ell_0,\ell_1,\ell_2,\ell_3))\in\mathbb{P}^3\times(\mathbb{P}^3)^*:\sum p_i\ell_i=0=\sum x_i\ell_i\right\}.$$

In other words, $P = (p_0, p_1, p_2, p_3)$ is "replaced" with the set of lines that go through P, which forms a projective plane $\mathbb{P}^2$ where P once was. The rest of the space is left alone.

Also the projective plane $\mathbb{P}^2$ blown up at two points (p_0, p_1, p_2) and (q_0, q_1, q_2) is

$$\left\{((x_0, x_1, x_2), (\ell_0, \ell_1, \ell_2), (h_0, h_1, h_2)) \in \mathbb{P}^2 \times (\mathbb{P}^2)^* \times (\mathbb{P}^2)^* : \sum \ell_i p_i = \sum h_i q_i = \sum x_i \ell_i = \sum x_i h_i = 0\right\}.$$

When we take $(p_0, p_1, p_2) = (0, 0, 1)$ and $(q_0, q_1, q_2) = (0, 1, 0)$, we have a Segre embedding into $\mathbb{P}^{11}$ and after modding out by the necessary relations we get the coordinate ring

$$k[x_0, x_1, \ldots, x_7]/(x_0x_1 + x_2x_5, x_0x_1 + x_3x_4, x_0x_2 + x_1x_7, x_0x_2 + x_3x_6).$$